Modeling power prices in competitive markets

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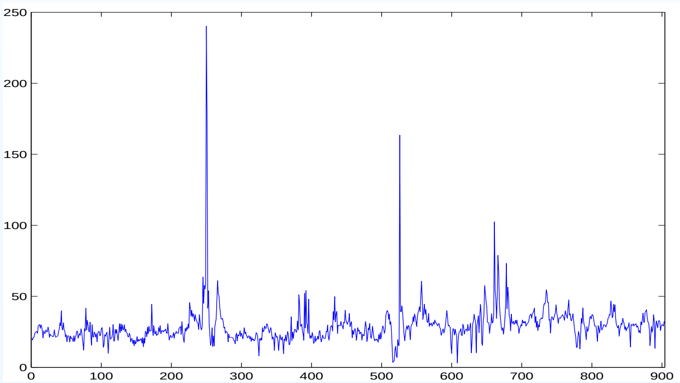
Outline

- Power prices dynamics: empirical analysis and stylized facts
- Reduced-form models
- Modeling spikes by excitable stochastic dynamics
- Hybrid models
  - Modeling the dynamics of the power margin
  - Modeling random switches in the supply curve
- Concluding remarks

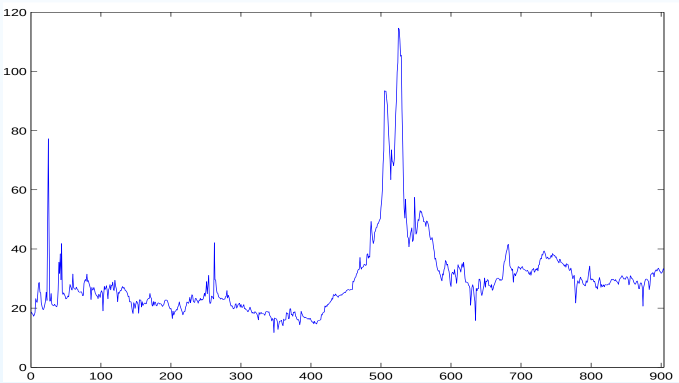
Historical behavior: prices

Daily spot prices: arithmetic average of the 24 market hourly prices

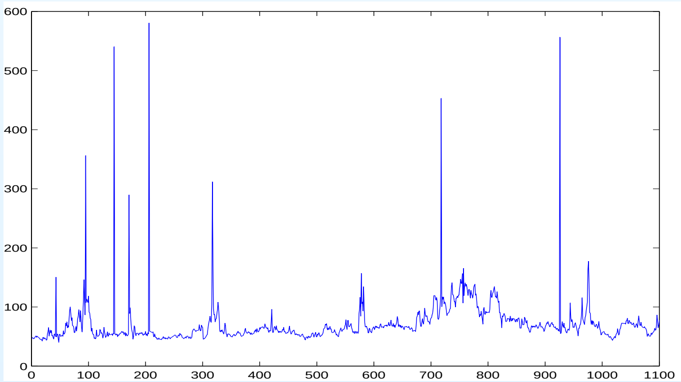
EEX: Jan 2, 01 - June 19, 04



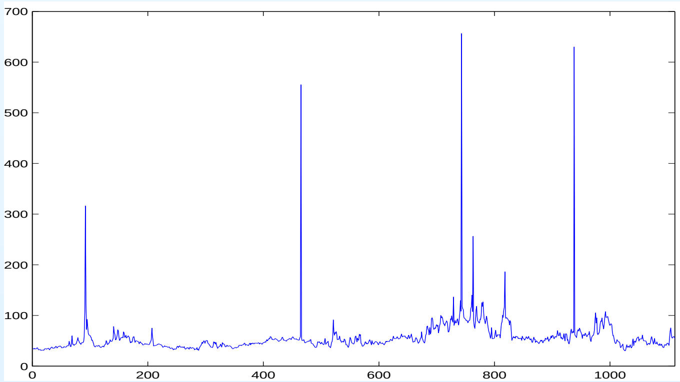
Nord Pool: Jan 2, 01 - June 19, 04



NEPOOL: Oct 22, 02 - Jan 15, 07



Texas: Oct 22, 02 - Jan 23, 07



Stylized facts

- Electricity prices are *variable* and *unpredictable* and show
  - **seasonality**
    - infradaily
    - weekly
    - annual
  - **multi-regime dynamics**: stable periods and turbulent periods
  - **mean-reversion**: prices fluctuate around the long run average
  - **jumps and short-lived spikes**: unparalleled upward jumps shortly followed by steep downward moves
  - **high volatility**: daily volatilities of about 30% are very frequent

Electricity price risk

prices randomness = electricity price risk

How to hedge this risk?

$$\Downarrow$$

FINANCE

A good representation of the spot price dynamics is crucial

- to value power derivatives
- to design supply contracts
- to define risk management strategies

Power options

The owner of a *call* option has the right (but not the obligation) to buy one megawatthour (MWh) of electricity at a future time T and at a fixed price K:

$$C_T = \max\{P_T - K, 0\}$$

- P_T : the price at time T of one MWh of electricity
- K : the *strike* price
- T : the *exercise* date

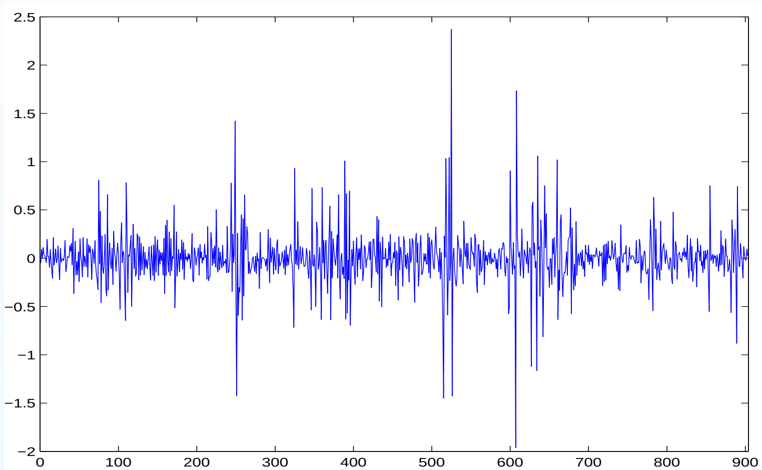
How to calculate the price at time t ($t < T$) of a call option?

$$\Downarrow$$

$$C_t = e^{-r(T-t)} E_t^* \left[\max\{P_T - K, 0\} \right]$$

Historical returns

EEX: Jan 2, 01 - June 19, 04



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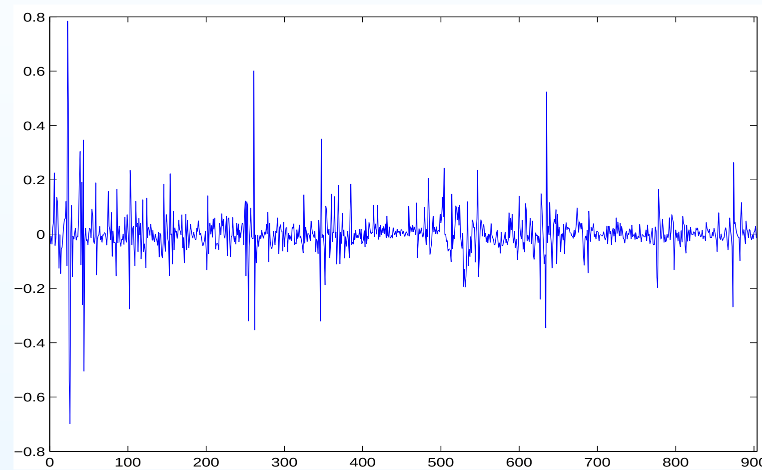

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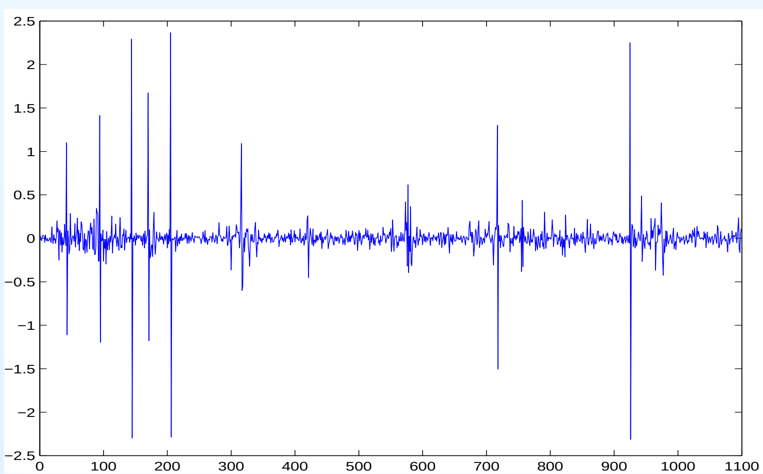

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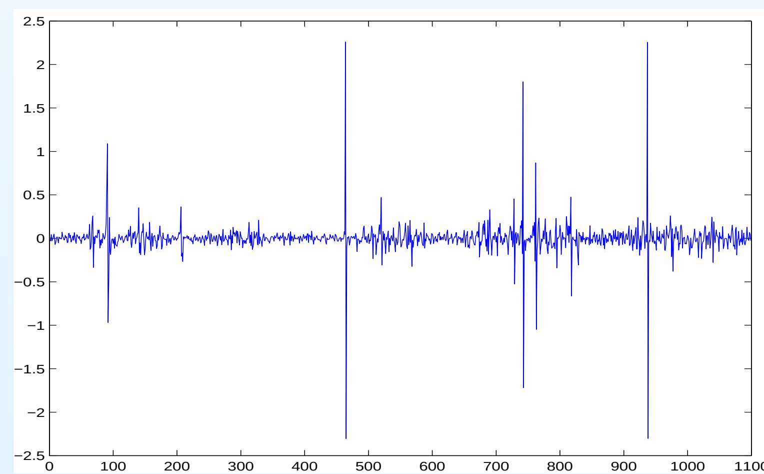
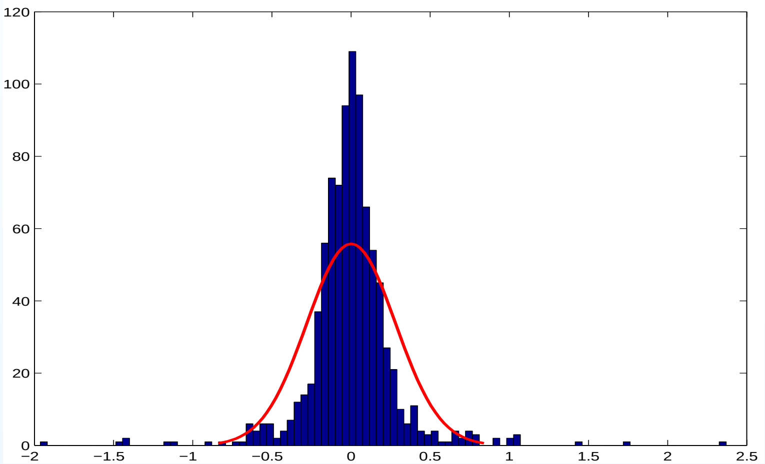
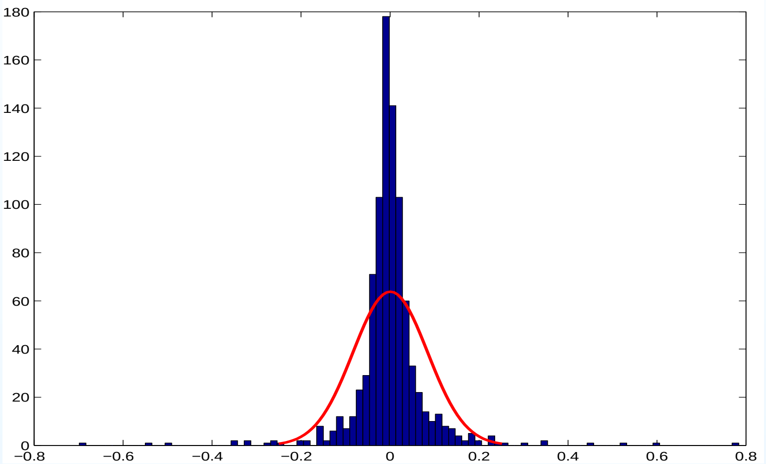


Empirical distributions

EEX: Jan 2, 01 - June 19, 04



Nord Pool: Jan 2, 01 - June 19, 04



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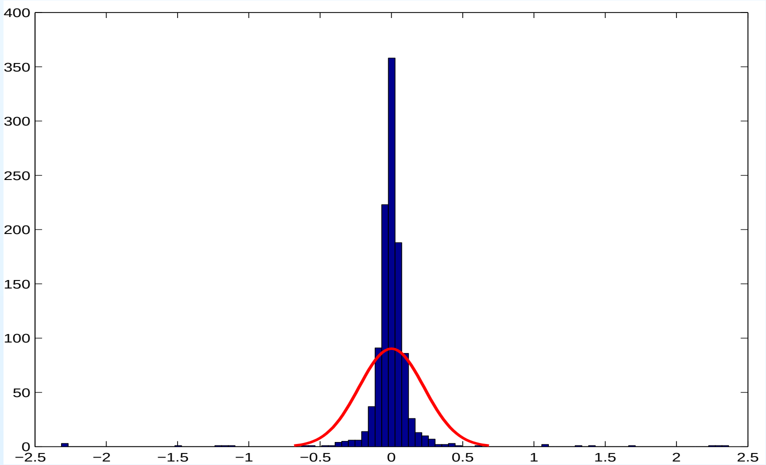
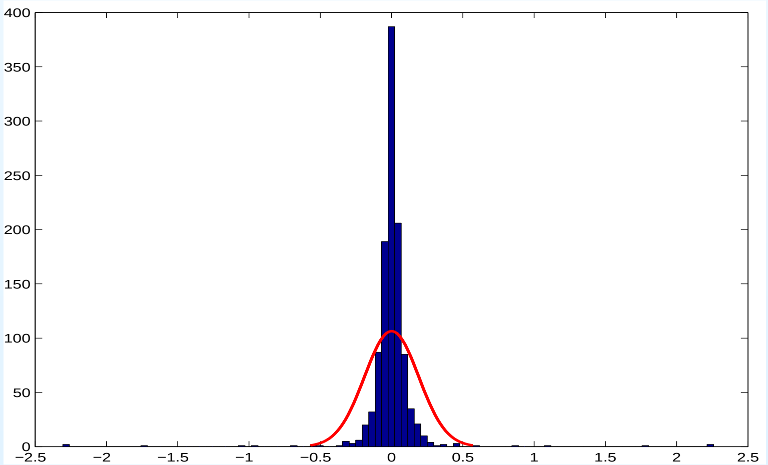


Texas: Oct 22, 02 - Jan 23, 07



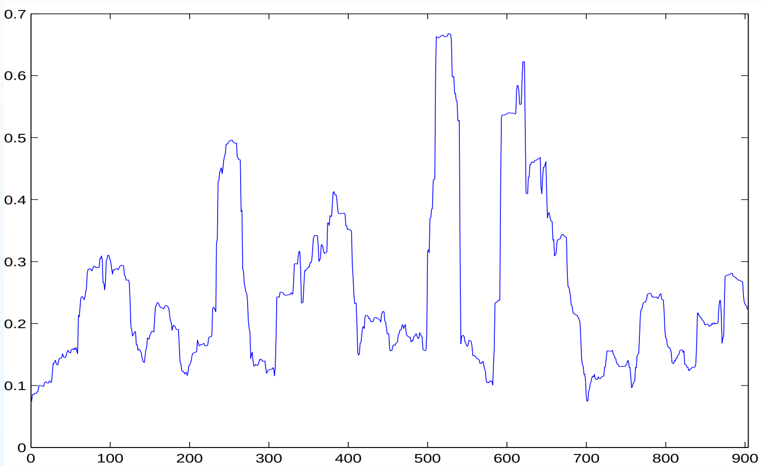
Descriptive statistics

The table displays descriptive statistics for log-returns

	EEX	Nord Pool	NEPOOL	Texas
Start	Jan 2, 01	Jan 2, 01	Oct 22, 02	Oct 22, 02
End	June 19, 04	June 19, 04	Jan 15, 07	Jan 26, 07
n	904	904	1103	1112
Min	-1.9627	-0.6983	-2.3140	-2.3058
Max	2.3694	0.7837	2.3657	2.2591
Mean	0.0005	0.0006	0.0008	0.0004
Std. dev.	0.2797	0.0837	0.2227	0.1900
Skew	0.4677	0.5440	0.3239	-0.1203
Kurt	16.8189	26.6337	66.8421	91.7385

Historical volatility

EEX: Jan 2, 01 - June 19, 04



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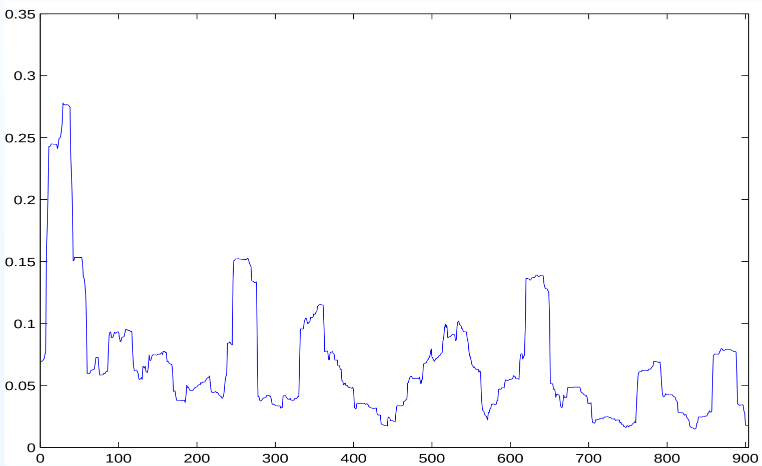

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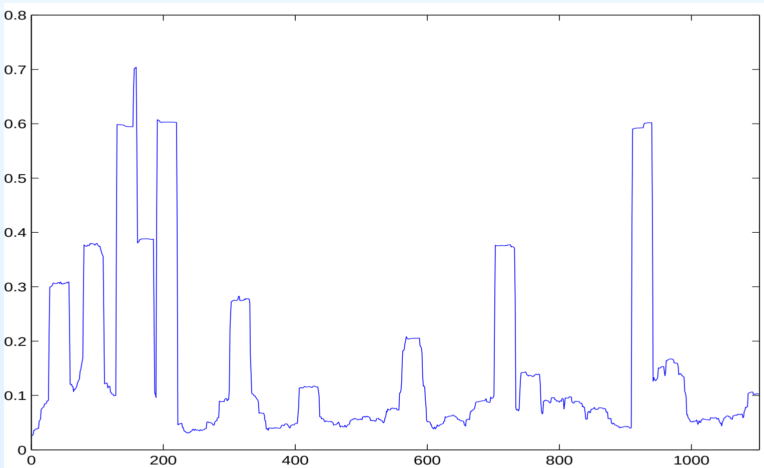
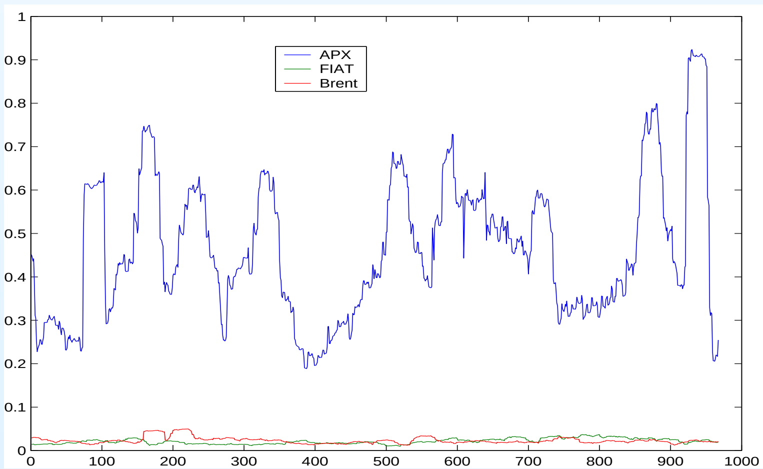


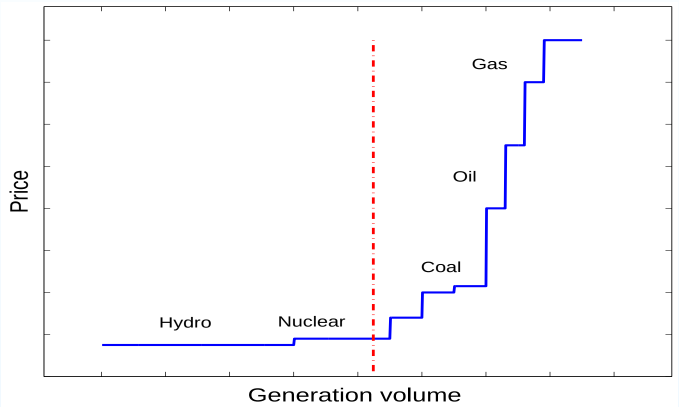
Comparing volatilities

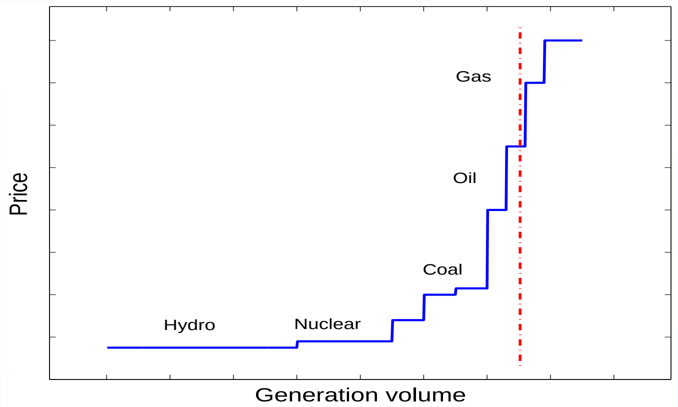


Electricity: a very special commodity

- Electricity cannot be stored
- electricity must be transported: some technical constraints must be satisfied (transmission capacity, frequency, voltage)
- the demand is highly inelastic and very sensitive to weather conditions (temperature)
- the production (supply) is characterized by
  - generators with low marginal costs to cover the base load (nuclear, hydro, and coal units)
  - generation units with high marginal costs to meet peak demand (oil-, gas-fired plants)

Understanding the spike phenomenon





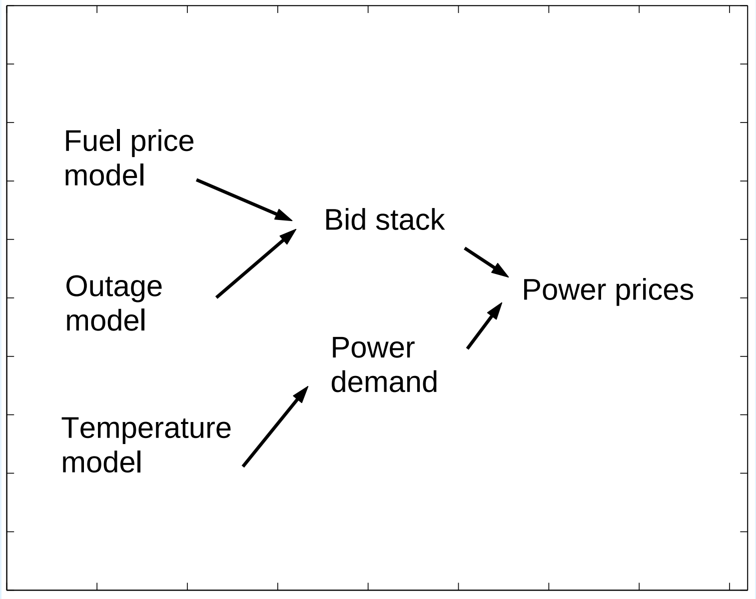
- supply curves exhibit a time variable kink after which offer prices rise almost vertically
- a spike may occur when the load curve is at the right of the kink
  - peaks in electricity demand
  - shortages in electricity generation (fluctuations of fuel prices, outages and grid congestions)
  - bidding strategies

Modeling electricity prices dynamics

- **Reduced-form models:** the objective is to replicate the statistical properties of electricity prices observed in real markets
  - stochastic models of electricity prices to capture:
    - seasonality
    - mean-reversion
    - spikes
    - high kurtosis
    - regime-switching
  - standard financial techniques for building risk-management strategies and pricing energy derivatives

Modeling electricity prices dynamics

- **Hybrid models:** the objective is to fuse the benefits of different modeling methodologies
  - *stochastic techniques* are used to describe the dynamics of the underlying drivers (temperature, fuel prices, outages..)
  - the *fundamental methodology* is used to represent demand-supply relations



Reduced-form models

- Two main classes of models:
  - **spot price models**
  - **forward price models**

- *Spot price models*
  - they provide a proper representation of the dynamics of spot prices
  - they do not allow the identification of the *market price of risk* when pricing derivatives

- *Forward price models*
  - they allow for pricing of derivatives in a straightforward way
  - they have some limitations as the inability to derive the features of spot prices from the analysis of forward curves

Spot models

We denote by

- P_t : the spot price at time t of one MWh of electricity
- $S_t = \ln P_t$

$$S_t = f_t + x_t$$

- f_t : highly predictable (deterministic) component accounting for seasonal effects
- x_t : random component reflecting unpredictable movements due to shortages in electricity generation and peaks in electricity demand

Mean-reverting diffusion models

The starting point of our analysis is the following model (Lucia and Schwartz 2002)

$$dx_t = -\alpha x_t dt + \sigma dw_t$$

- w_t : a standard Brownian motion

Remark:

- high values of the mean-reversion rate and of the volatility parameter
- jumps are not allowed
- the volatility is constant
- statistical properties of simulated trajectories are not consistent with empirical data

Mean-reverting jump-diffusion models

$$dx_t = -\alpha x_t dt + \sigma dw_t + J_t dN_t$$

- w_t : a standard Brownian motion
- N_t : a Poisson jump process

$$\Downarrow$$

- signed jumps (Escribano *et al.* 2002)
- reverting jumps (Geman and Roncoroni 2006)
- alternating positive and negative jumps (Weron *et al.* 2004)

Remark: high values of the mean-reversion rate

Markov regime-switching models

- Regime-switching models can be useful
  - to capture the nonlinearities of the prices dynamics
  - to distinguish the normal stable motion from the spike regime
  - to introduce various mean-reversion rate and volatilities depending on the state of the system
- Different regimes correspond to different sectors of the stack curve
- The switching mechanism between the states is governed by an unobservable Markov process

A three-regime model

- To separate the *stable motion* from the *spike regime*, the following approach has been proposed (Huisman and Mahieu 2003)

$$dx_t = \begin{cases} -\alpha_0 x_t dt + \sigma_0 dw_{0t} & \text{stable regime} \\ \mu_1 dt + \sigma_1 dw_{1t} & \text{the spike regime} \\ -\alpha_{-1} x_t dt + \sigma_{-1} dw_{-1t} & \text{back to the stable regime} \end{cases}$$

- The transition probabilities matrix: the duration of the spikes is of "one day"

$$\pi = \begin{pmatrix} 1-\gamma dt & 0 & 1 \\ \gamma dt & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

A two-regime model

- The following model (Mari 2006, Physica A) tends to distinguish the stable motion and the "turbulent" dynamics

$$dx_t = \begin{cases} -\alpha_0 x_t dt + \sigma_0 dw_{0t} & \text{stable regime} \\ -\alpha_1 x_t dt + \sigma_1 dw_{1t} + J dN_t & \text{"turbulent" regime} \end{cases}$$

- The spike duration can be more than one day: the probability transition matrix is

$$\pi = \begin{pmatrix} 1 - \gamma dt & \eta dt \\ \gamma dt & 1 - \eta dt \end{pmatrix}$$

Modeling spikes by excitable dynamics

- A dynamical system is *excitable* when a stationary solution is stable with regard to perturbations smaller than a characteristic threshold. If it is perturbed above this threshold, the system performs a large cycle, coming back to its initial state

- The FitzHugh-Nagumo (FHN) model can be cast in the following form

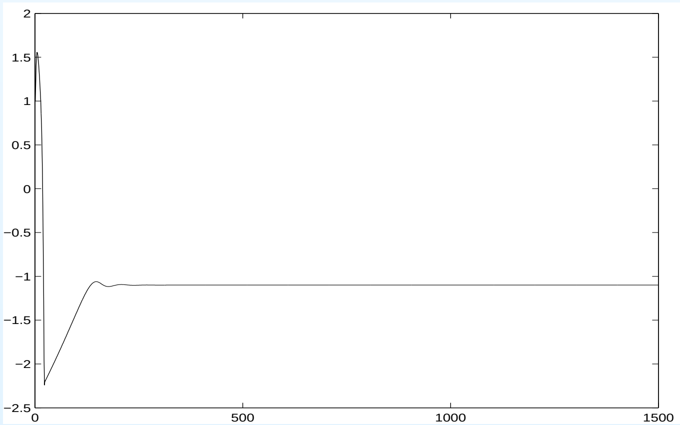
$$\varepsilon \dot{x}_t^s = x_t^s - \frac{{x_t^s}^3}{3} - y_t, \qquad \varepsilon > 0$$
$$\dot{y}_t = x_t^s + a$$

- The FHN-dynamics is proposed for illustrative purposes, more general maps can be used to describe spiky systems

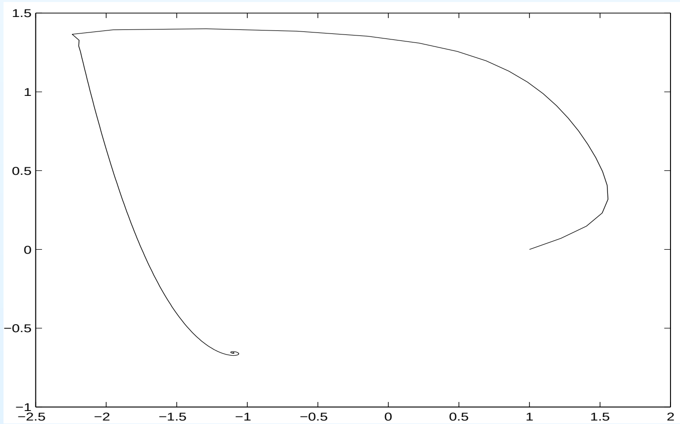
Stability properties: the excitable dynamics

- only one fixed point $(-a, a^3/3 - a)$
- the parameter of bifurcation a determines the behavior of the system: at the bifurcation point $a = 1$ the stability of the fixed point changes
- for $a > 1$ the fixed point is stable: the model shows an *excitable* behavior

Stable motion ($\varepsilon = 0.1$, $a = 1.1$)



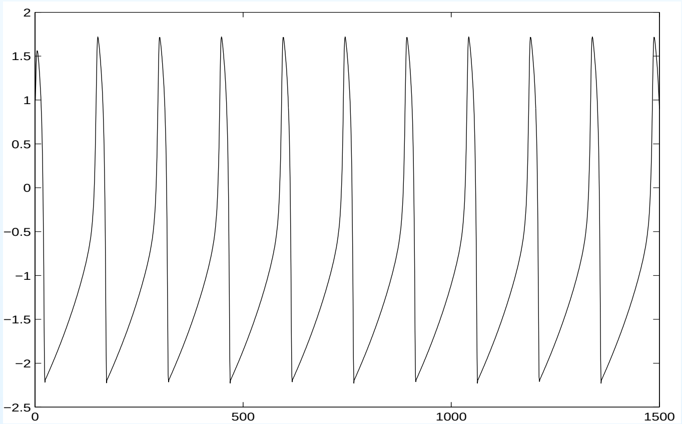
Stable motion: phase-space



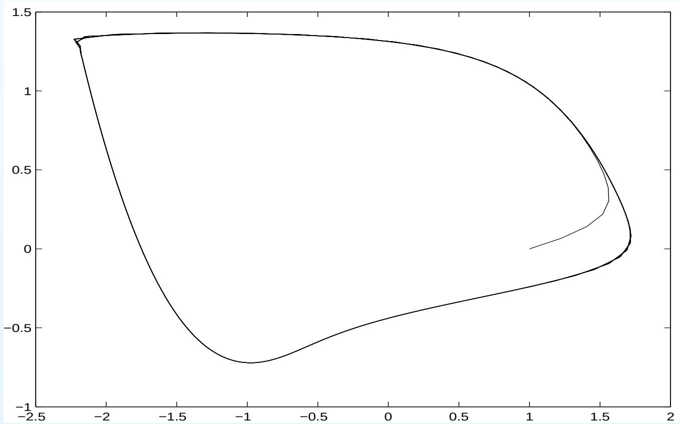
Stability properties: the oscillatory dynamics

- for $a < 1$ the fixed point is unstable but a limit cycle exists: the model shows an *oscillatory* behavior

Oscillatory dynamics ($\varepsilon = 0.1$, $a = 0.986$)

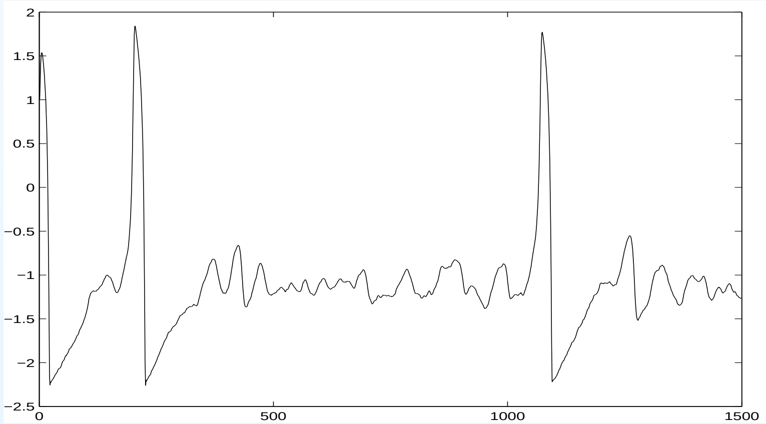


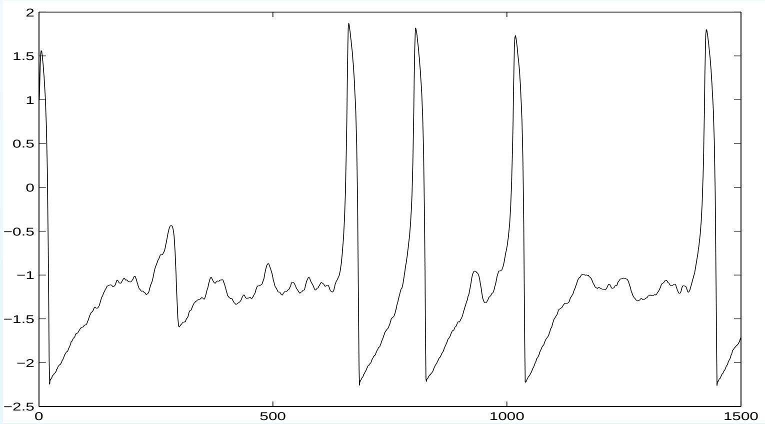
Oscillatory dynamics: phase space



An excitable stochastic dynamics

$$\varepsilon dx_t^s = (x_t^s - \frac{x_t^{s\,3}}{3} - y_t)dt$$
$$dy_t = (x_t^s + a)dt + \sigma_s dw_t$$





The noise, w_t, induces fluctuations in the bifurcation parameter: the system switches between stable and unstable motion thus allowing for stochastic spikes formation ($\varepsilon = 0.1$, $a = 1.1$, $\sigma_s = 0.08$)

Modeling electricity prices: a two regime-switching model

- The stable motion is governed by a mean-reverting diffusion process
- the spike regime is described by a stochastic FHN dynamics

$$\begin{cases} dx_t = (\mu_0 - \alpha_0 x_t)dt + \sigma_0 dw_{0t} \\ x_t = \phi(x_t^s + \psi) \end{cases}$$

ϕ, ψ : two additional parameters

(De Sanctis and Mari 2007, Physica A)

Transitions between regimes

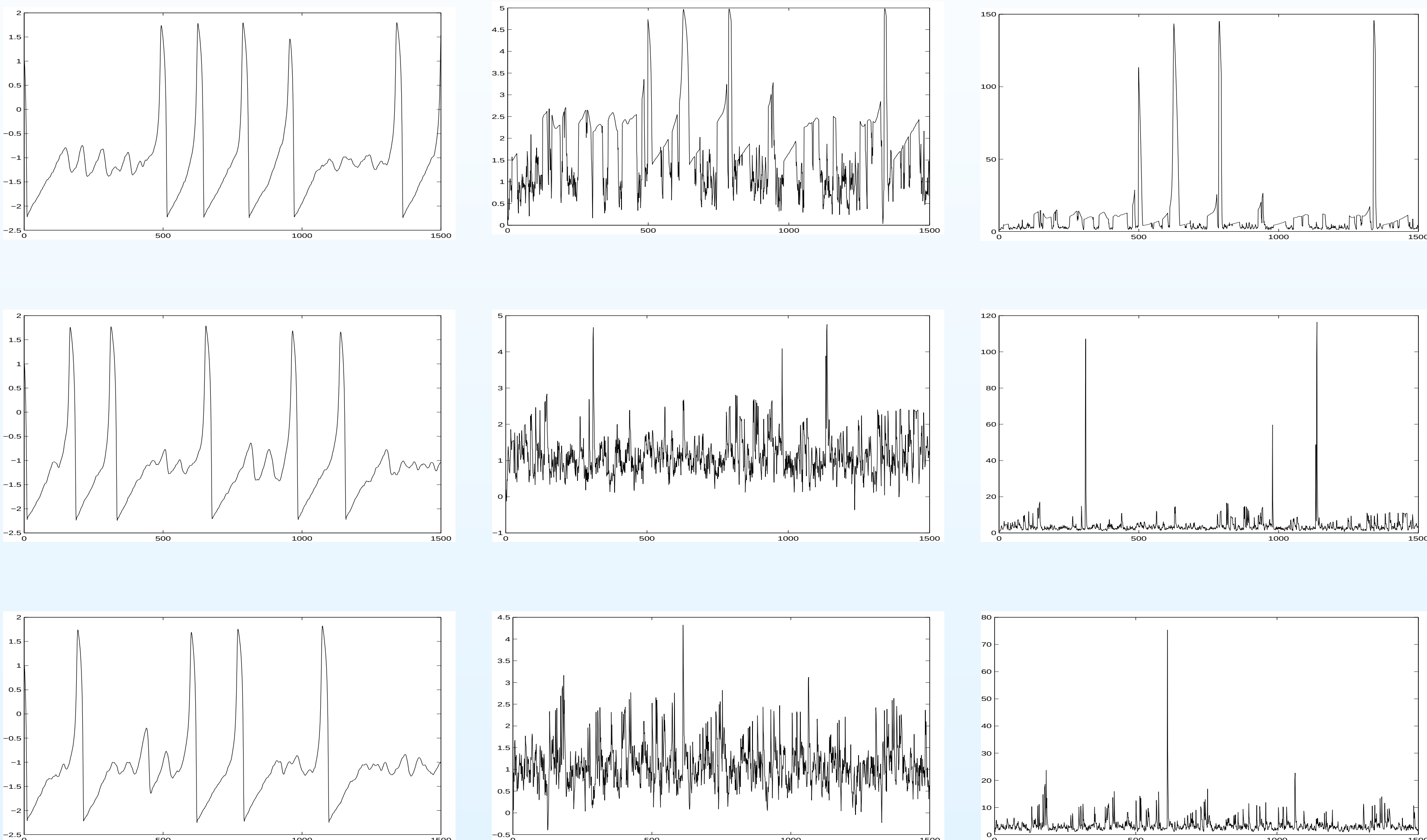
The transition probabilities matrix can be cast in the following form

$$\pi = \begin{pmatrix} 1-\gamma dt & 1-\eta dt \\ \gamma dt & \eta dt \end{pmatrix}$$

- the probability to remain in the stable regime as well the probability to revert back to the stable regime are "high"
- the duration of spikes can be more than one day
- although the stochastic oscillations in the FHN-dynamics have the same height, in our model the height and the duration of the spikes can be controlled by the parameters γ and η

The spike dynamics

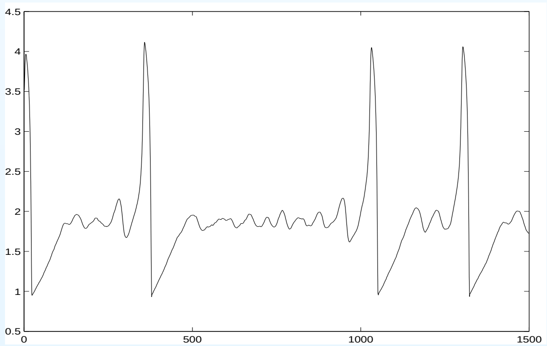
x^s | x | $z = e^x$

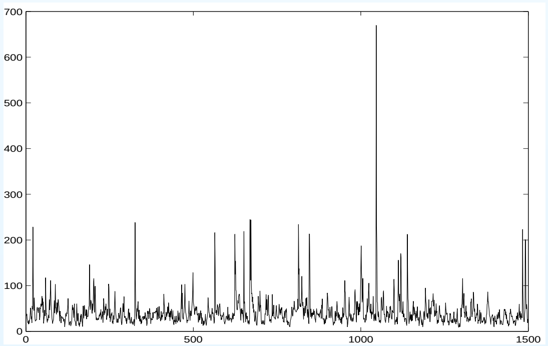


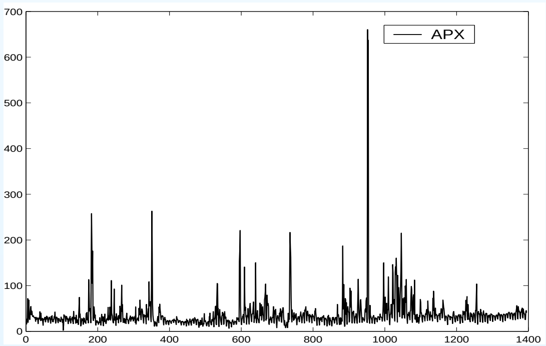
The probability of remaining in the spike regime is assumed to be of 95%, 50%, and 10%

Simulating prices paths: the APX market

Seasonality	Stable dynam.	Spikes dynam.	
$\mu = 2.63$	$\mu_0 = 0.30$	$\varepsilon = 0.10$	$\phi = 0.89$
$\beta_1 = 0.35$	$\alpha_0 = 0.33$	$a = 1.05$	$\psi = 3.80$
$\beta_2 = 0.65$	$\sigma_0 = 0.25$	$\sigma = 0.08$	$\xi dt = 0.015$
			$\eta dt = 0.075$







5000 paths	Mean	Std. dev.	Skewness	Kurtosis
Observed	0.0005	0.4633	0.7116	8.8401
Simul.	0.0005	0.4647	1.2094	8.8837

A three-regime extension

Modeling electricity prices using a single mean-reverting regime produces very high values of the mean-reversion rate

$$\begin{cases} dx_t = \left(\mu_0 - \alpha_0 x_t\right) dt + \sigma_0 dw_{0t} \\ x_t = \phi\left(x_t^S + \psi\right) \\ dx_t = -\alpha_{-1} x_t dt + \sigma_{-1} dw_{-1t} \end{cases}$$

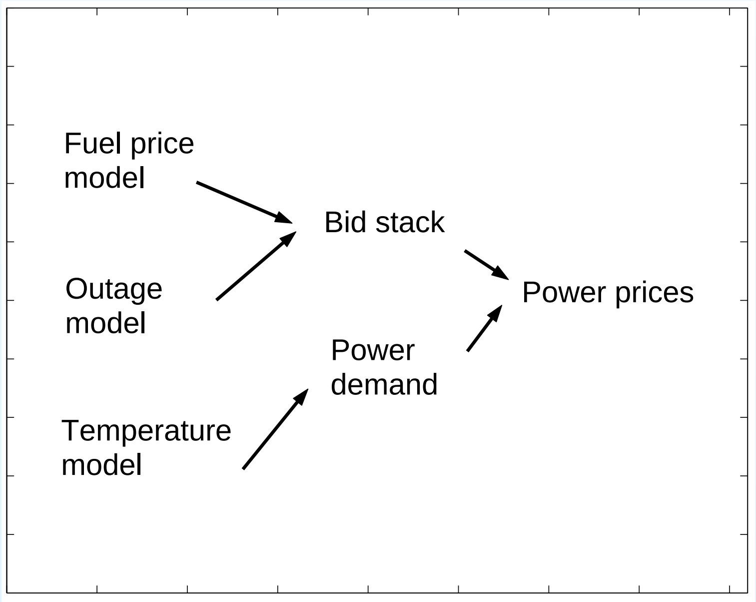
- different regimes are characterized by different values of mean-reversion parameters and volatilities
- α_{-1} is responsible of the strong prices reduction after a spike

Hybrid models

Reduced form models are not capable of incorporating non-price information

$$\Downarrow$$

hybrid models: fundamental and market data



A demand-supply approach

- The supply curve in the (q_t, P_t)-plane can be cast in the following functional form (Barlow, 2002)

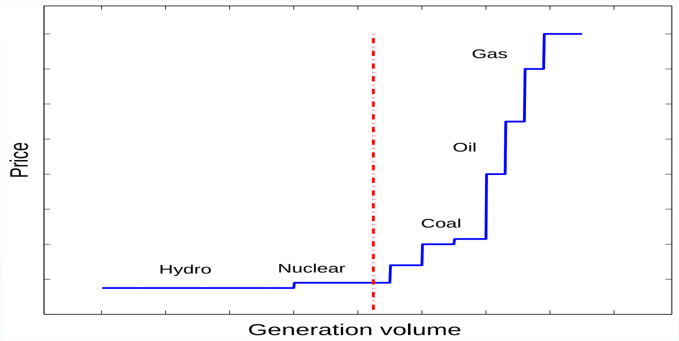
$$P_t = \begin{cases} \left[(a - q_t)/b\right]^{1/c} & if \quad q_t < a - \varepsilon b \\ \varepsilon^{\frac{1}{c}} & if \quad q_t \geq a - \varepsilon b \end{cases}$$

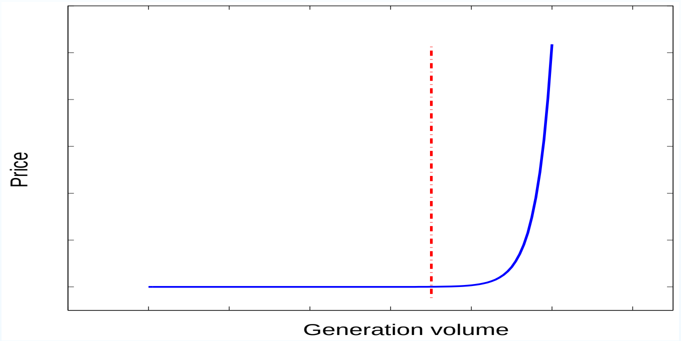
- the demand is very inelastic

$$q_t = D_t$$

  where D_t is a stochastic process independent of the power price
- D_t is driven by a mean-reverting diffusion process
- a fixed nonlinear supply function is used to produce spikes in the simulated trajectories

Modeling the dynamics of the power margin





- The (time-variable) supply curve in the (q_t, P_t)-plane is assumed to be (Mari 2008, Journal of Energy Markets)

$$P_t = h_0 \exp\left(\frac{q_t - k_t}{h_1}\right)$$

k_t is a stochastic process defining the kink position at time t, and h_0, h_1 are normalization parameters

- $q_t = D_t$, where D_t is independent of the power price

Equilibrium between supply and demand

$$P_t = h_0 \exp\left(\frac{D_t - k_t}{h_1}\right) \equiv h_0 \exp(-z_t)$$

where

$$z_t = \frac{k_t - D_t}{h_1}$$

is the (normalized) **power margin** at time t

$$\Downarrow$$

$\boxed{\text{electricity prices experience spikes when } z_t < 0}$

Modeling the dynamics of z_t

$$z_t = f_t + x_t$$

- f_t : highly predictable (deterministic) component accounting for seasonal effects
- x_t : random component reflecting unpredictable changes in the power margin level (shortages in electricity generation and peaks in electricity demand)
  - jump-diffusion dynamics
  - regime-switching dynamics

Empirical results: a two-regime model

$$dx_t = \begin{cases} -\alpha_0 x_t dt + \sigma_0 dw_{0t} \\ -\alpha_1 x_t dt + \sigma_1 dw_{1t} + J dN_t \end{cases}$$

Maximum likelihood estimation by the Hamilton filtering technique

	EEX	Nord Pool	NEPOOL	Texas
α_0	0.1858 (0.030)	0.0056 (0.005)	0.0197 (0.008)	0.0173 (0.007)
σ_0	0.1246 (0.005)	0.0223 (0.001)	0.0482 (0.002)	0.0371 (0.002)
α_1	0.4719 (0.075)	0.0306 (0.013)	0.2216 (0.037)	0.0781 (0.019)
σ_1	0.3617 (0.036)	0.0730 (0.006)	0.1397 (0.008)	0.1083 (0.007)
λdt	0.0641 (0.040)	0.1218 (0.033)	0.085 (0.022)	0.0488 (0.012)
σ_J	1.0914 (0.346)	0.2805 (0.046)	1.2119 (0.187)	1.1209 (0.186)
$1-\gamma dt$	0.9408 (0.016)	0.8935 (0.021)	0.9476 (0.011)	0.9208 (0.017)
$1-\eta dt$	0.8206 (0.046)	0.8635 (0.034)	0.8710 (0.029)	0.9040 (0.028)
LL	-2757.1	-1584.7	-3441.5	-3107.9

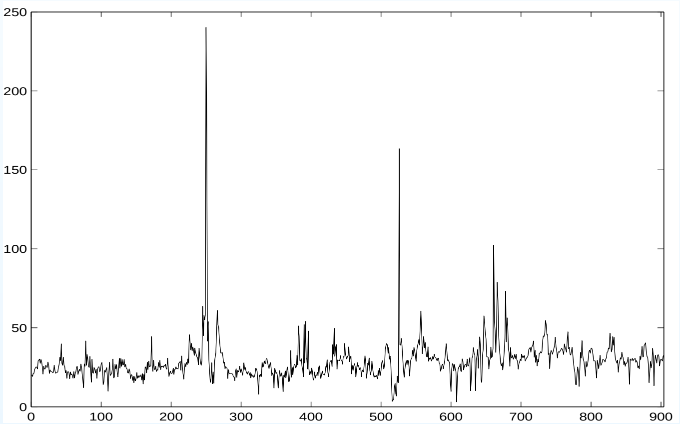
Statistical properties

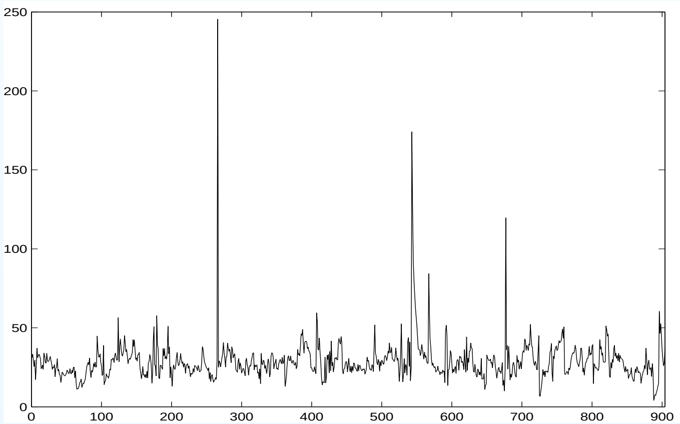
		Mean	Std. dev.	Skewness	Kurtosis
EEX	Observed	0.0005	0.2797	0.4677	16.8189
	Simul.	0.0003	0.2774	0.0140	17.0803
		(0.0004)	(0.0285)	(0.8714)	(7.5577)
Nord Pool	Observed	0.0006	0.0837	0.5440	26.6337
	Simul.	0.0003	0.0828	0.0473	23.5910
		(0.0005)	(0.0077)	(1.1351)	(6.3148)
NEPOOL	Observed	0.0008	0.2227	0.3239	66.8421
	Simul.	0.0004	0.2145	0.0100	68.7083
		(0.0004)	(0.0329)	(2.8303)	(22.3961)

Averages over 5000 simulated paths

A simulated trajectory

Historical behavior of prices (left) and a simulated path (right) at EEX





	Mean	Std.dev.	Skewness	Kurtosis
Observed	28.7095	13.1889	7.6585	103.5765
Simulated	28.7405	13.2254	7.5184	103.3850

Random switches in the supply-curve

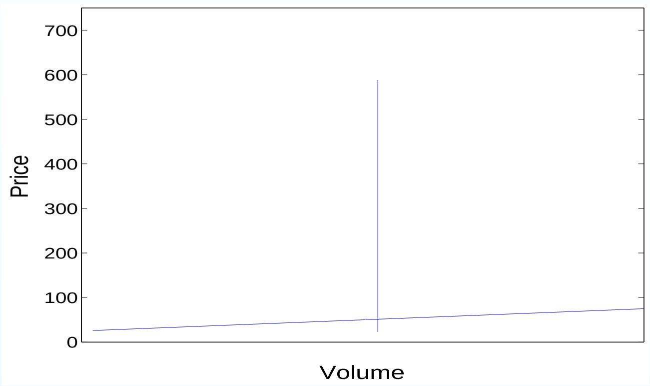
We assume (Mari 2010, Physica A) that the functional form of the offer curve is given by

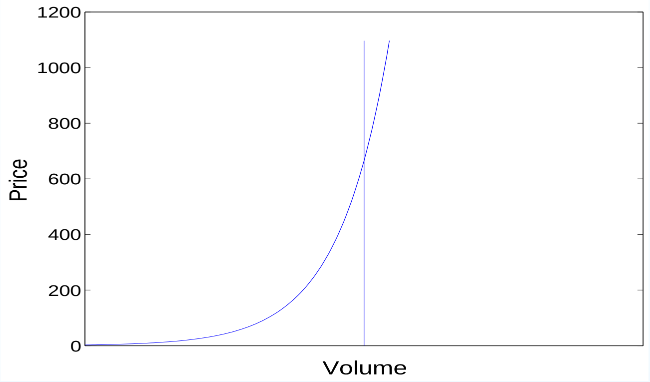
$$P_t = h_{0t} \left(\frac{q_t}{a_t} \right)^{\beta_t}$$

- h_{0t} : highly predictable (deterministic) component accounting for seasonal effects
- a_t : random component accounting for random movements of the offer curve
- β_t : a discrete, strictly positive Markov process assuming only two values, β_0 and β_1, with $\beta_0 < \beta_1$ and $\beta_1 > 1$, and transition matrix

$$\pi = \begin{pmatrix} 1 - \gamma dt & 1 - \eta dt \\ \gamma dt & \eta dt \end{pmatrix}$$

Random switches in the supply-curve





The equilibrium between demand and supply is assured if

$$P_t = h_{0t} \left(\frac{D_t}{a_t} \right)^{\beta_t}$$

β_t and a_t capture random movements of the offer curve due to unpredictable changes in the generation process as outages, grid congestions as well as random power generation of renewable sources

Market prices dynamics

Posing

- $\log P_t = \log h_{0t} + p_t, \quad \log D_t = \log f_{xt} + x_t,$
  $\log a_t = \log f_{xt} + y_t$

we get

$$\boxed{p_t = \beta_t(x_t - y_t).}$$

In the transition from state i to state j during the infinitesimal time interval $[t, t+dt]$ the dynamics of power prices is given by

$$\boxed{dp_t = \beta_j\left[dx_t - dy_t\right] + \frac{\beta_j - \beta_i}{\beta_i} p_t.}$$

A mean-reverting model

If drift coefficients are assumed to be linear functions of x and y

$$\begin{cases} dx = (a_{ij}x + b_{ij}y)dt + \sigma^x_{ij} dw^x_{ij} + H^x_{ij} \\ dy = (c_{ij}x + d_{ij}y)dt + \sigma^y_{ij} dw^y_{ij} + H^y_{ij} \end{cases}$$

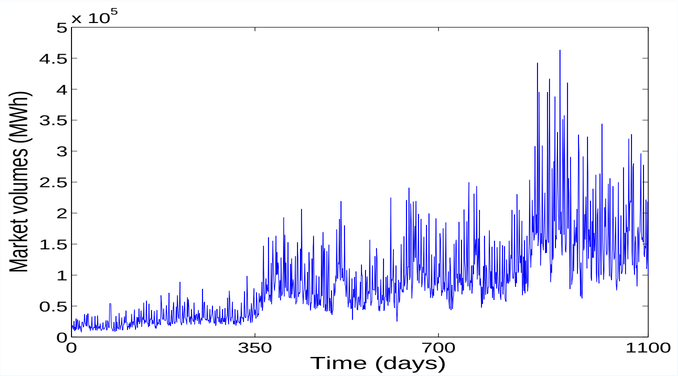
where H^x_{ij} and H^y_{ij} account for non-Brownian random movements as jumps and spikes

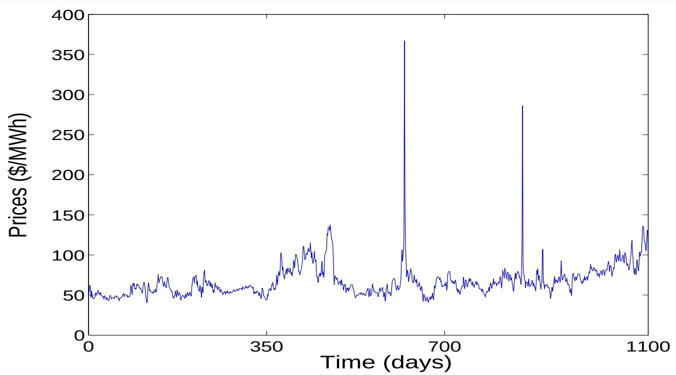
$$\Downarrow$$

the dynamics of the model can be reformulated in terms of market prices and traded volumes in the following way

$$\begin{cases} dx = -\xi^p_{ij} pdt - \xi^x_{ij} xdt + \sigma^x_{ij} dw^x_{ij} + H^x_{ij} \\ dp = -\alpha^p_{ij} pdt - \alpha^x_{ij} xdt + \sigma^p_{ij} dw^p_{ij} + H^p_{ij} + \frac{\beta_j - \beta_i}{\beta_i} p \end{cases}$$

A two-regime specification



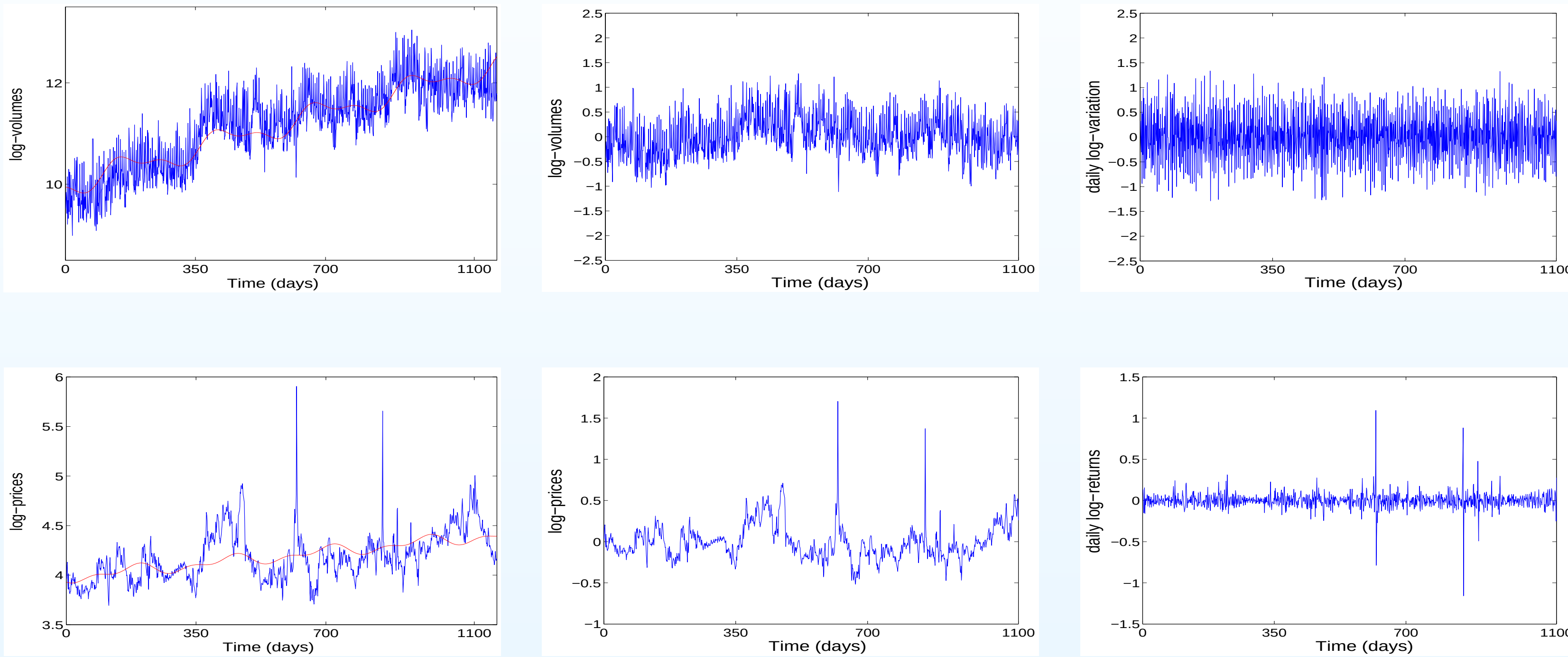


$$dx_t = -\xi p_t dt - \xi^\dagger x_t dt + \sigma^x dw_t^x .$$

$$dp(t) = \begin{cases} -\alpha_0 p_t dt - \alpha_0^\dagger x_t dt + \sigma_0^p dw_{0t}^p \\ -\alpha_1 p_t dt + \sigma_1^p dw_{1t}^p + J dN_t , \end{cases}$$

- J is normally distributed with 0-mean and standard deviation σ_J
- w_t^x, w_{0t}^p, w_{1t}^p, and the Poisson process are mutually independent and independent of the jump amplitude

Empirical analysis: log-volumes and log-prices



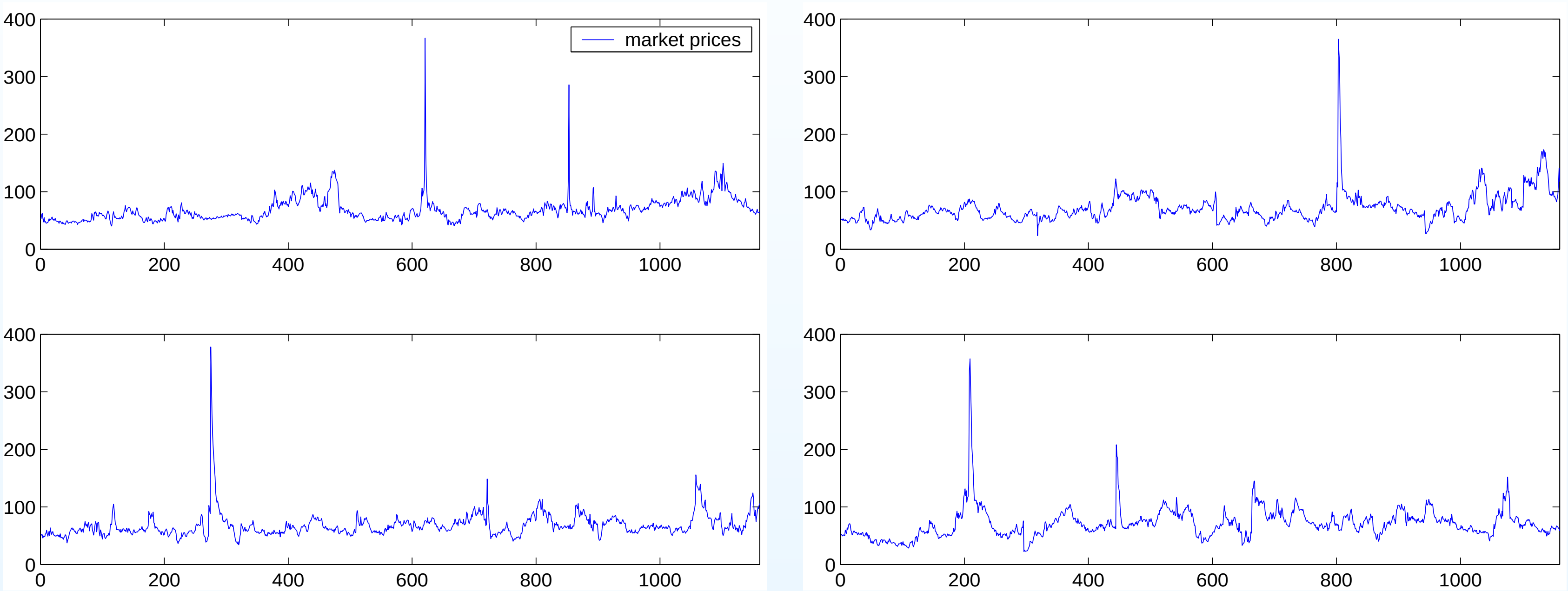
	Volumes	Prices
Mean	-0.0003	-0.0002
Std. Dev.	0.5083	0.0957
Skewness	-0.0462	0.1230
Kurtosis	2.6300	47.0006

Empirical results

Maximum likelihood estimation by the Hamilton filtering technique (Hamilton 1989)

Volumes		Prices	
ξ	-0.1849 (0.056)	α_0	0.0349 (0.010)
$\xi^\dagger$	0.7093 (0.028)	$\alpha_0^\dagger$	-0.0155 (0.004)
σ^x	0.4066 (0.009)	σ_0^p	0.0455 (0.002)
		α_1	0.0969 (0.029)
		σ_1^p	0.0982 (0.006)
		λ	0.0329 (0.014)
		σ_J	0.5738 (0.125)
		$1-\gamma$	0.9457 (0.014)
		$1-\eta$	0.8986 (0.026)
LL_x	-602.0	LL_p	1472.9

Empirical results



5000 paths	Volumes	Prices
Mean	0.0003 (0.0004)	-0.0001 (0.0002)
Std.dev.	0.5068 (0.0117)	0.0937 (0.0106)
Skewness	-0.0042 (0.0688)	0.0483 (2.3356)
Kurtosis	2.9976 (0.1468)	43.6457 (22.3458)

Concluding remarks: modeling primary factors

- Anderson (2004): a four modules specification
  - forced outages
  - planned outages
  - load
  - prices

- Eydeland and Wolyniec (2003): a six modules specification
  - fuel prices (futures prices)
  - forced and planned outages
  - generation stack
  - temperature
  - demand
  - spot prices: a bid stack transformation of the demand

Concluding remarks: pricing power derivatives

- To make the models realistic, more and more fundamentals factors can be incorporated
  - highly complex hybrid models are subject to a significant **modeling risk**

- balance between model parsimony and adequacy to capture the main characteristics of prices

$$\Downarrow$$

$$\boxed{\textbf{pricing energy derivatives}}$$